

Basics

- $(a+b)^2 = a^2 + b^2 + 2ab$
- $(a-b)^2 = a^2 + b^2 - 2ab$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$
- $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$
- $a^2 - b^2 = (a+b)(a-b)$
- $a^2 + b^2 = (a+b)^2 - 2ab$
OR
 $(a-b)^2 + 2ab$
- $(a+b)^2 = (a-b)^2 + 4ab$
- $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$
- $\sin^2 \theta + \cos^2 \theta = 1$
- $1 + \tan^2 \theta = \sec^2 \theta$
- $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$
- $\sin^{-1} a \pm \sin^{-1} b = \sin^{-1} (a\sqrt{1-b^2} \pm b\sqrt{1-a^2})$
- $\cos^{-1} a \pm \cos^{-1} b = \cos^{-1} (ab \mp \sqrt{1-a^2}\sqrt{1-b^2})$
- $\tan^{-1} a \pm \tan^{-1} b = \tan^{-1} \left(\frac{a \pm b}{1 \mp ab} \right)$
- $\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$$\bullet \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\bullet \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\bullet \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\bullet \cos C - \cos D = 2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$\bullet \sin 3A = 3 \sin A - 4 \sin^3 A$$

$$\bullet \cos 3A = 4 \cos^3 A - 3 \cos A$$

$$\bullet \sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \bullet \cos 2A &= \cos^2 A - \sin^2 A \\ &= 1 - 2 \sin^2 A \\ &= 2 \cos^2 A - 1 \end{aligned}$$

$$\bullet 1 + \cos 2A = 2 \cos^2 A$$

$$\bullet 1 - \cos 2A = 2 \sin^2 A$$

$$\bullet \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

Relation and function

* Function

$f: A \rightarrow B$ is said to be a function if it satisfies any one of the two given condition :-

1. Every element in set A should have its corresponding image in set B .
2. Two different domain can have one range

NOTE: $f: A \rightarrow B$ is not a function if one domain has two different range.

$$f = (a, b) \quad \begin{array}{l} a = \text{domain} \\ b = \text{range} \end{array}$$

- Let A & B are two sets such that set A contains x elements, set B contains y elements. Then total no. of function is :-

$$\begin{aligned} f: A \rightarrow B &= y^x \\ f: B \rightarrow A &= x^y \\ f: A \rightarrow A &= x^x \\ f: B \rightarrow B &= y^y \end{aligned}$$

- Types of function



One-One Function

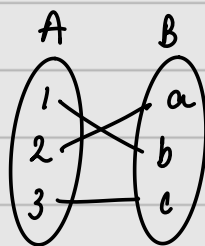
(Injective function)

A function $f: A \rightarrow B$ is said to be one one if every element in set A should have its corresponding different image in Set B .

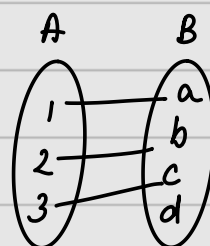
(subject to Condition)

Moreover $f: A \rightarrow B$ is said to be one one if $f(x_1) = f(x_2)$

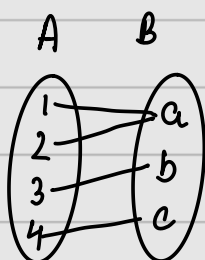
$$\Rightarrow x_1 = x_2$$



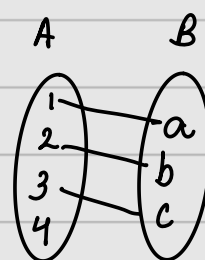
one - one



one - one



not one - one



not one - one

NOTE: If a set A contains m elements and set B contains n elements then total no. of one one function :-

$f: A \rightarrow B$ is

$$\begin{cases} n^m & \text{if } n \geq m \\ 0 & \text{if } n < m \end{cases}$$

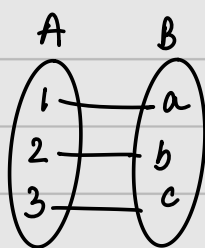


Onto function

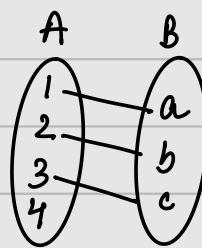
(surjective function)

A function is said to be onto if every element of set B has at least one pre image in A . (subject to condition)

Moreover $f(x)$ is said to be onto if co-domain = range of $f(x)$

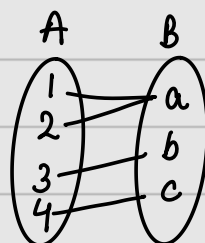


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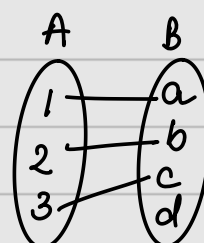
Onto

one one



onto
not one one

not one one



not onto
one one

NOTE: Let A & B are two sets containing m & n no. of elements then total number of onto function from A to B =

$$\begin{cases} \sum_{r=1}^n (-1)^{n-r} {}^nC_r {}^mH^r, & \text{if } m \geq n \\ 0, & \text{if } m < n \end{cases}$$

* Special Case:-

1. If $n=2$ and $m \geq 2$ then the number of onto function from A to B is $2^m - 2$.
2. If $n=3$ and $m \geq 3$, then the number of onto functions from A to B is $3^m - 3(2^m - 1)$

NOTE: Let A & B are two sets $f: A \rightarrow B$ is said to be bijjective function if it is one one & onto.

Let A & B are two sets, set A contains m elements, set B contains n elements. Total no. of bijective function $f: A \rightarrow B = m!$ if $m=n$
 0 if $m \neq n$

• Inverse of a function

If a function is bijective then it is invertible, i.e. its inverse exists. eg.

$$y = f(x)$$

$$x = f^{-1}(y)$$

* Relation

• Cartesian Pair

Let A & B are two sets. Set A contains m elements and set B contains n elements then total number of cartesian pair.

$$A \text{ to } B = m \times n$$

$$B \text{ to } A = n \times m$$

$$A \text{ to } A = m \times m$$

$$B \text{ to } B = n \times n$$

• Types of Relation:

Reflexive relation	$a R a$	} Equivalence relation satisfies all three relations
Symmetric Relation	$a R b \Rightarrow b R a$	
Transitive Relation	$a R b, b R c \Rightarrow a R c$	

* If a set A contains 3 elements then total number of equivalence relation = 5

If a set A contains 4 elements then total number of equivalence relation = 15

• Identity relation is the smallest equivalence relation.

• All the cartesian pairs are universal relation. Universal relation is the largest equivalence relation.

* Let A is a set having n elements then total number of reflexive relation is $2^{n(n-1)}$ and total number of symmetric relation is $2^{\frac{n(n+1)}{2}}$

Inverse Trigonometry

*

domain

range

$\sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	$\mathbb{R}$	$]-\pi/2, \pi/2[$
$\cot^{-1} x$	$\mathbb{R}$	$]0, \pi[$
$\sec^{-1} x$	$\mathbb{R} -]-1, 1[$	$[0, \pi] - \{\pi/2\}$
$\operatorname{cosec}^{-1} x$	$\mathbb{R} -]-1, 1[$	$[-\pi/2, \pi/2] - \{0\}$

* $\sin^{-1} x \neq \frac{1}{\sin x}$

* $\sin \pi = 0$ $\cos \pi = -1$ $\tan \pi = 0$	* $\sin \theta = x \Rightarrow \sin x = \theta$ $\cos \theta = x \Rightarrow \cos x = \theta$ $\tan \theta = x \Rightarrow \tan x = \theta$	* $\sin^{-1}(\sin x) = x$ $\cos^{-1}(\cos x) = x$ $\tan^{-1}(\tan x) = x$
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- $\sin(\pi - x) = \sin x$
- $\cos(\pi + x) = -\cos x$
- $\cos(2\pi + x) = \cos x$
- $\cos(\pi/2 + x) = -\sin x$
- $\sin(-x) = -\sin x$
- $\cos(-x) = \cos x$
- $-\cos x = \text{no change}$
- $\sin 2A = 2 \sin A \cos A$
- $\cos 2A \Rightarrow 1 - 2 \sin^2 A$

= odd + x = negative
 = even + x = positive

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

- $\tan A/2 = \sqrt{\frac{1 - \cos A}{1 + \cos A}}$
- $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$
- $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

$$* \quad \begin{aligned} \sin^{-1} x + \cos^{-1} x &= \pi/2 \\ \tan^{-1} x + \cot^{-1} x &= \pi/2 \\ \sec^{-1} x + \operatorname{cosec}^{-1} x &= \pi/2 \end{aligned}$$

$$* \quad \begin{aligned} \operatorname{cosec}^{-1} x &= \sin^{-1} 1/x \\ \sec^{-1} x &= \cos^{-1} 1/x \\ \tan^{-1} x &= \cot^{-1} 1/x \end{aligned}$$

$$* \quad \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$$

$$\cdot \quad \tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy} \right)$$

$$\cdot \quad 2\tan^{-1} x \Rightarrow \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

$$\Rightarrow \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$\Rightarrow \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$$

$$\cdot \quad \tan^{-1} \left(\frac{1-x}{1+x} \right) = \tan^{-1} 1 - \tan^{-1} x$$

$$\cdot \quad \tan^{-1} \left(\frac{1+x}{1-x} \right) = \tan^{-1} 1 + \tan^{-1} x$$

$$* \quad \sin^{-1} x + \sin^{-1} y = \sin^{-1} [x\sqrt{1-y^2} + y\sqrt{1-x^2}]$$

$$\cdot \quad \sin^{-1} x - \sin^{-1} y = \sin^{-1} [x\sqrt{1-y^2} - y\sqrt{1-x^2}]$$

$$\cdot \quad \cos^{-1} x + \cos^{-1} y = \cos^{-1} [xy - \sqrt{1-y^2} \cdot \sqrt{1-x^2}]$$

$$\cdot \quad \cos^{-1} x - \cos^{-1} y = \cos^{-1} [xy + \sqrt{1-y^2} \cdot \sqrt{1-x^2}]$$

$$\cdot \quad \sin^{-1} \frac{a}{x} + \sin^{-1} \frac{b}{x} = \frac{\pi}{2}, \text{ then } x = \sqrt{a^2 + b^2}$$

$$\cdot \quad \sin^{-1} \frac{a}{x} + \sin^{-1} \frac{b}{x} = -\frac{\pi}{2}, \text{ then } x = -\sqrt{a^2 + b^2}$$

$$\cdot \quad 1 + \cos 2A = 2\cos^2 A$$

$$\cdot \quad 1 - \cos 2A = 2\sin^2 A \quad \Rightarrow \quad 1 - \cos A = 2\sin^2 A/2$$

$$\cdot \quad \sin 2A = 2\sin A \cos A \quad \Rightarrow \quad \sin A = 2\sin A/2 \cos A/2$$

Matrices and determinants

- $A = []_{R \times C}$. where $R = \text{rows}$ and $C = \text{columns}$.
eg. $A = \begin{bmatrix} 3 & 2 & 4 \\ 7 & 8 & 6 \end{bmatrix}_{2 \times 3}$

- Total elements = [rows $\times$ columns] orders.

13 elements can be written in $\{1 \times 13 \text{ and } 13 \times 1\} = 2$ orders.

- $i = \text{rows}$; $j = \text{columns}$

NOTE: Using P no. of elements, total matrix of order $m \times n$ that can be formed is given by $(P)^{m \times n}$.

eg. Using elements $(1, 2, 3)$ how many total matrix of order 2×2 can be formed?

$$\Rightarrow (3)^{2 \times 2} \Rightarrow 3^4 \Rightarrow \underline{\underline{81}} \text{ Ans.}$$

* **Trace of a matrix** = sum of the elements of the diagonals.

eg. trace of $\begin{bmatrix} 1 & 3 & 5 \\ 4 & 6 & 2 \\ 3 & 9 & 1 \end{bmatrix} \Rightarrow 1 + 6 + 1 \Rightarrow \underline{\underline{8}} \text{ Ans}$

* **Formation of a matrix**

eg. $A = (3 \times 2)$ matrix ; element = $a_{ij} = \frac{3i + 2j}{2}$

$$\Rightarrow A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 5/2 & 7/2 \\ 4 & 5 \\ 11/2 & 13/2 \end{bmatrix}$$

$$a_{11} = \frac{3+2}{2} = \frac{5}{2}$$

$$a_{12} = \frac{3+4}{2} = \frac{7}{2}$$

$$a_{21} = \frac{6+2}{2} = 4$$

$$a_{22} = \frac{6+4}{2} = 5$$

$$a_{31} = \frac{9+2}{2} = \frac{11}{2}$$

$$a_{32} = \frac{9+4}{2} = \frac{13}{2}$$

* Addition and Subtraction of a Matrix

* rows and columns of both the matrix should be equal!

⇒ just add/subtract.

eg. $A = \begin{bmatrix} 3 & 2 \\ 4 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$; find $A+B$ and $A-B$

$$\Rightarrow A+B = \begin{bmatrix} 3-1 & 2+2 \\ 4+3 & -5-4 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 4 \\ 7 & -9 \end{bmatrix}$$

$$\Rightarrow A-B = \begin{bmatrix} 3+1 & 2-2 \\ 4-3 & -5+4 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix}$$

* Equality of two matrix.

* elements of both the matrix will be totally equal!

eg. $\begin{bmatrix} 3x+2y & 2 \\ 4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 3x-y \\ 3x+y & 2 \end{bmatrix}$

$$\Rightarrow \begin{array}{r} 3x+2y = 5 \\ -3x+y = -4 \\ \hline y = 1 \end{array}$$

$$\& \quad \begin{array}{l} x+y = 2 \Rightarrow x+1 = 2 \\ \Rightarrow x = 1 \end{array}$$

* Multiplication of a matrix

* $AB \neq BA$

⇒ for multiplication, no. of column of first matrix should be equal to the no. of rows of the second matrix.

eg. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 5 \end{bmatrix}$

in this case, only AB is possible, i.e. BA is not possible.

$AB \Rightarrow$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \times B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 1 \\ 2 & 1 & 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} (1 \times 2) + (2 \times 1) + (3 \times 2) & (1 \times 3) + (2 \times 2) + (3 \times 1) & (1 \times 4) + (2 \times 1) + (3 \times 5) \\ (2 \times 2) + (1 \times 1) + (5 \times 2) & (2 \times 3) + (1 \times 2) + (5 \times 1) & (2 \times 4) + (1 \times 1) + (5 \times 5) \end{bmatrix}$$

$$AB = \begin{bmatrix} 2+2+6 & 3+4+3 & 4+2+15 \\ 4+1+10 & 6+2+5 & 8+1+25 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 10 & 10 & 21 \\ 15 & 13 & 34 \end{bmatrix}$$

$$AB = 2 \times 3$$

NOTE:

If order of matrix $A = m \times n$ and order of matrix $B = p \times q$ then order of matrix AB will be.

$$\begin{array}{ccc} A & B & \\ m \times n & p \times q & \Rightarrow AB \\ & & m \times q. \end{array}$$

NOTE:

$$\text{If } A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}; \text{ then } A^n = 2^{n-1}(A)$$

* For word problem (BOARDS 2025 question).

Three shopkeepers Gaurav, Rizwan, and Jacob use carry bags made of polythene, handmade paper, and newspaper.

The numbers of bags used by Gaurav, Rizwan, and Jacob are (20, 30, 40), (30, 40, 20), and (40, 20, 30) for polythene, handmade paper, and newspaper bags respectively. The costs of these bags are ₹1, ₹5, and ₹2 each. Gaurav, Rizwan, and Jacob spend ₹A, ₹B, and ₹C respectively.

The questions to answer using matrices and determinants are:

(i) Represent the information in Matrix form.

(ii) Find the values of ₹A, ₹B, and ₹C.

$$\begin{bmatrix} 20 & 30 & 40 \\ 30 & 40 & 20 \\ 40 & 20 & 30 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 250 \\ 270 \\ 200 \end{bmatrix} \begin{array}{l} A \\ B \\ C \end{array}$$

Quantity $\rightarrow$
Cost $\downarrow$

* Identity matrix

* always fixed.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} ; I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

NOTE: $I^n = I$ whereas AI or $IA = A$

* Transpose of a matrix

If any matrix is of order $m \times n$, then its transpose denoted as A^T or A' will be of order $n \times m$.

$$\text{eg. } A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

- NOTE:
- $(AB)^T = B^T A^T$
 - $(BA)^T = A^T B^T$
 - $(A+B)^T = A^T + B^T$
 - $(A-B)^T = A^T - B^T$
 - $(A^T)^T = A$

- NOTE:
- If $\sin x = \sin \alpha$
 $x = n\pi + (-1)^n \alpha$
 - If $\cos x = \cos \alpha$
 $x = 2n\pi \pm \alpha$

* Symmetric matrix

When $A = A^T$; then it is said to be a symmetric matrix.

$$\text{eg. } A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} ; A^T = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} ; \text{ here } A = A^T$$

$\therefore A$ is symmetric

* Skew Symmetric Matrix

when A is any matrix and $A^T = -A$, then A is skew symmetric.

eg. $A = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$; $A^T = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$; $-A = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$

SPECIAL NOTE: The diagonals of all skew symmetric matrix is equal to 0.

- NOTE:**
1. If A is any matrix, then $(A + A^T)$ is a symmetric matrix.
 2. If A is any matrix, then $(A - A^T)$ is a skew symmetric matrix.
 3. If a constant k is multiplied to a symmetric matrix, then the resulting matrix so formed will also be a symmetric matrix.
 4. If a constant k is multiplied to a skew symmetric matrix, then the resulting matrix so formed will also be a skew symmetric matrix.
- * 5. A matrix can be expressed as a sum of symmetric & skew symmetric matrix & can be written as -

$$\frac{1}{2} (A + A^T) + \frac{1}{2} (A - A^T) = A$$

* Diagonal matrix

These matrix are whose principle diagonal elements are different and rest all the elements are zero.

eg. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

* Scalar Matrix

These matrix are whose principle diagonal elements are same & rest all the elements are zero.

eg. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

NOTE :

All Identity matrices are Scalar matrices
but not all Scalar matrices are Identity matrices.

* Upper Triangular matrix

A square matrix is called upper triangular matrix if $A_{ij} = 0$ for all $i > j$, that is for all the elements below the principle diagonal is zero.

eg. $A = \begin{bmatrix} 2 & 1 & 5 \\ 0 & 3 & 6 \\ 0 & 0 & 4 \end{bmatrix}$

* Lower Triangular matrix

A square matrix is called lower triangular matrix if $A_{ij} = 0$ for all $i < j$, that is for all the elements above the principle diagonal is zero.

eg. $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 5 & 6 & 4 \end{bmatrix}$

* Minor

eg. $A = \begin{bmatrix} 5 & 3 \\ 7 & 2 \end{bmatrix} \Rightarrow \begin{matrix} m_{11} = 2 & m_{21} = 3 \\ m_{12} = 7 & m_{22} = 5 \end{matrix}$

$\therefore \text{minor} = \begin{bmatrix} 2 & 7 \\ 3 & 5 \end{bmatrix}$

* Cofactor

$\Rightarrow C_{ij} = (-1)^{i+j} \cdot M_{ij}$

* Adjoint A = Transpose of Cofactor of A

* $A \cdot \text{Adjoint } A = |A| I$ or determinant of A [adjoint A]

* Inverse of a matrix

If A is any matrix then its reciprocal i.e. $1/A$ is written as A^{-1} .

A^{-1} is called the inverse of matrix A .

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adjoint } A$$

If $|A| = 0$, $A^{-1} = \infty$, matrix is not invertible. Inverse of matrix A does not exist.

* Solving a linear equation using matrix method/rule.

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

* Determinants:~

Value of a matrix is called a determinant. Determinants can be found only of square matrices, i.e. rows = columns.

determinants = $|A|$; where A = matrix.

eg. $A = \begin{bmatrix} 3 & 6 \\ 7 & 5 \end{bmatrix} \Rightarrow |A| = (3 \times 5) - (6 \times 7) \Rightarrow 15 - 42 \Rightarrow -27$

* Properties of determinants -

i. Singular matrix / matrix having no inverse. i.e. $|A| = 0$

eg. $A = \begin{bmatrix} 9 & 6 \\ 3 & 2 \end{bmatrix} \Rightarrow |A| = (9 \times 2) - (6 \times 3) \Rightarrow 18 - 18 \Rightarrow 0.$

ii. If any 2 rows or 2 columns of a determinant are identical then the value of determinant is equal to 0.

eg. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$; $|A|$ will be 0
 $\because$ row 1 and row 3 is same.

eg. $A = \begin{bmatrix} a & a & u \\ b & b & y \\ c & c & z \end{bmatrix}$; $|A|$ will be 0
 $\because$ column 1 and column 2 is same.

iii. If something is common in particular row or column of a determinant then that common can be written outside.

eg. $A = \begin{vmatrix} 2 & 9 & 1 \\ 4 & 7 & 5 \\ 6 & 3 & 8 \end{vmatrix} \Rightarrow A = 2 \begin{vmatrix} 1 & 9 & 1 \\ 2 & 7 & 5 \\ 3 & 3 & 8 \end{vmatrix}$

eg. $A = \begin{vmatrix} 2 & 3 & 1 \\ 4 & 6 & 7 \\ 6 & 9 & 1 \end{vmatrix} \Rightarrow A = 2 \times 3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 7 \\ 3 & 3 & 1 \end{vmatrix} \Rightarrow 6 \times 0 \Rightarrow 0.$

iv. two adjacent rows or two adjacent columns can be interchanged by putting minus sign outside the determinant.

eg. $A = \begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} \Rightarrow (-) \begin{vmatrix} b & e & h \\ a & d & g \\ c & f & i \end{vmatrix} \Rightarrow (-)(-) \begin{vmatrix} b & e & h \\ c & f & i \\ a & d & g \end{vmatrix}$

v. A determinant can be expressed as a sum of 2 determinants.
(determinants can be broken but not added)

eg. $\begin{vmatrix} a & x+y & c \\ b & y+z & d \\ c & z+x & e \end{vmatrix}$

$\Rightarrow \begin{vmatrix} a & x & c \\ b & y & d \\ c & z & e \end{vmatrix} + \begin{vmatrix} a & y & c \\ b & z & d \\ c & x & e \end{vmatrix}$

vi. If A is a square matrix, then $|A^n| = |A|^n$

if $A = \begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix}$, find $|A^5|$

$\Rightarrow |A|^5 \quad (\because |A^n| = |A|^n)$

$\Rightarrow (12-10)^5 = 2^5$
 $= \underline{\underline{32}} \text{ Ans.}$

vii. If A is a square matrix, such that $|A| = a$ then $|A^{-1}| = \frac{1}{a}$

viii. If A and B are 2 sq. matrix such that AB exists, then
 $|AB| = |A||B|$

ix. If A is a sq. matrix of order n then $|A| = |A^T|$

v.v.v.
imp

- x. If A is a matrix of order n then $|KA| = K^n |A|$
- xi. If A is a matrix of order n then determinants of $|\text{Adj } A| = |A|^{n-1}$
- xii. If all the elements of a particular row or column of a determinant are 0, then the value of determinant = 0.
- xiii. If possible before expanding the determinant, try to make 2 elements of a particular row/column zero.
- Additive Inverse + sum of 2 matrix is zero.

* Area of Triangle

if ABC is a Δ with co-ordinates $a(x_1, y_1)$, $b(x_2, y_2)$, $c(x_3, y_3)$
then area of Δ is given by:-

$$\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

NOTE: if area of Δ is 0, the points are said to be collinear, they all lie on the same straight line.

Differentiation

* Formulae :-

$$1.) \frac{d}{dx} x^n = nx^{n-1}$$

$$2.) \frac{d}{dx} \log x = \frac{1}{x}$$

$$3.) \frac{d}{dx} a^x = a^x (\log a)$$

$$4.) \frac{d}{dx} e^x = e^x$$

$$5.) \frac{d}{dx} e^{-x} = -e^{-x}$$

$$6.) \frac{d}{dx} x = 1$$

$$7.) \frac{d}{dx} \frac{1}{x} = -\frac{1}{x^2}$$

$$8.) \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$9.) \frac{d}{dx} k = 0 \quad (k \text{ is constant})$$

$$10.) \frac{d}{dx} \sin x = \cos x$$

$$11.) \frac{d}{dx} \cos x = -\sin x$$

$$12.) \frac{d}{dx} \tan x = \sec^2 x$$

$$13.) \frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$14.) \frac{d}{dx} \sec x = \sec x \tan x$$

$$15.) \frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

$$16.) \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$17.) \frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$$

$$18.) \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$19.) \frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$$

$$20.) \frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$$

$$21.) \frac{d}{dx} \operatorname{cosec}^{-1} x = \frac{-1}{x\sqrt{x^2-1}}$$

* log formulae

- $\log_a x = \frac{\log x}{\log a}$
- $\log x^m = m \log x$
- $e^{\log x} = x$

* Product Rule

- $y = u \cdot v$

$$\frac{dy}{dx} = u \cdot \frac{d}{dx} v + v \frac{d}{dx} u$$

* Quotient rule

- $y = \frac{u}{v}$

$$\frac{dy}{dx} = \frac{v \cdot \frac{d}{dx} u - u \frac{d}{dx} v}{v^2}$$

* Differentiation of function of a function OR Chain rule.

solve from outside to inside and multiply them. [NOT ADD BUT MULTIPLY].

* Basic trigonometric formula

- $\sin 2x = 2 \sin x \cos x$; $1 = \sin^2 x + \cos^2 x$
 $\therefore 1 + \sin 2x = (\sin^2 x + \cos^2 x + 2 \sin x \cos x)$ OR $(\sin x + \cos x)^2$
- $\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$
- $\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$
- $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$
- $\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$

* Logarithm formula

- $\log x^m = m \log x$
- $\log xy = \log x + \log y$
- $\log \frac{x}{y} = \log x - \log y$
- if $\log x = y$, then $x = e^y$

* Infinite series

- $y = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \dots}}}}$
 $y = \sqrt{x + y}$

* differentiation of one function w.r.t another function

solve the first function as $\frac{dy}{dx}$ and the second function as $\frac{dx}{dt}$

* differentiation of parametric function

solve x as $\frac{dx}{dt}$ and y as $\frac{dy}{dt}$ and then finally $= \frac{dy}{dx} \times \frac{dt}{dx}$

* Higher order derivative

$$y = f(x) = x^6$$

$$\frac{dy}{dx} = f'(x) = y_1 = y' = 6x^5$$

$$\frac{d^2y}{dx^2} = f''(x) = y_2 = y'' = 30x^4$$

Continuity

and

Differentiability

* Formula:~ (Continuity)

A function affects is said to be continuous and $x=a$ if $f(a)^+ = f(a) = f(a)^-$

$$\text{where } f(a)^+ = \lim_{h \rightarrow 0} f(a+h)$$

$$f(a)^- = \lim_{h \rightarrow 0} f(a-h)$$

* Formula:~ (Differentiability)

If the function is differentiable then it is continuous but if a function is continuous it may or may not be differentiable.

A $f(x)$ is said to be differentiable at $x=a$ if RHD (Right Hand derivative) = LHD (Left hand derivative)

$$RHD = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

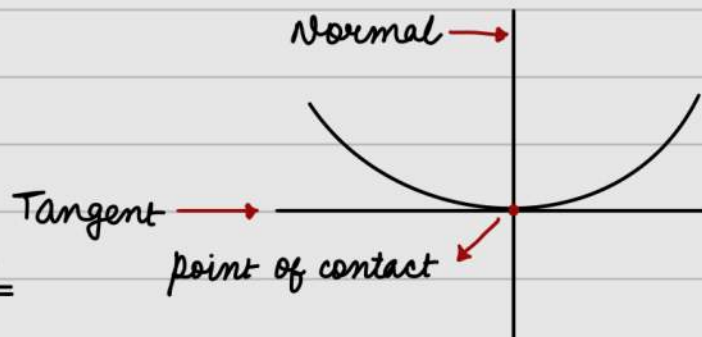
$$LHD = \lim_{h \rightarrow 0} \frac{f(a-h) - f(a)}{-h}$$

Tangent and Normal

* Tangent and Normal

Tangent is the line which touches the curve at a single point. This point is called the Point of contact denoted by (x_1, y_1)

Normal is a line which is perpendicular to the Tangent & passes through the point of contact.



* Equation of Tangent

$$\frac{y - y_1}{x - x_1} = \left. \frac{dy}{dx} \right|_{x_1, y_1}$$

where (x_1, y_1) is the point of contact and $\left. \frac{dy}{dx} \right|_{x_1, y_1}$ is the slope of tangent.

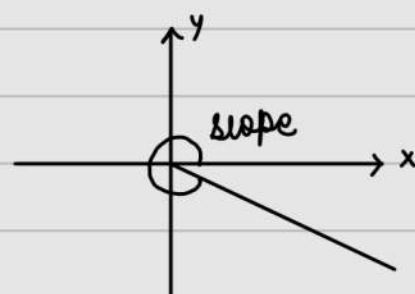
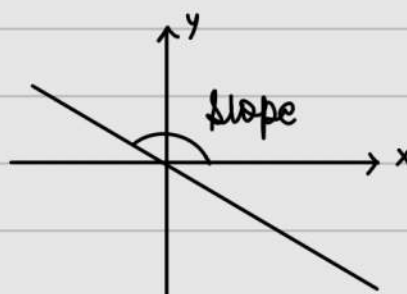
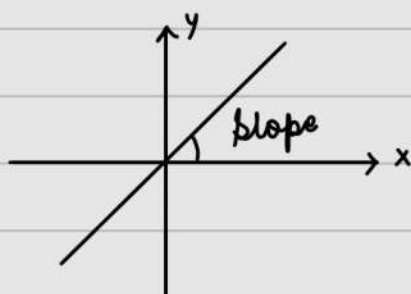
* Equation of Normal

$$\frac{y - y_1}{x - x_1} = -\frac{1}{\left. \frac{dy}{dx} \right|_{x_1, y_1}}$$

where (x_1, y_1) is the point of contact and $-\frac{1}{\left. \frac{dy}{dx} \right|_{x_1, y_1}}$ is the slope of normal.

* NOTE:

Slope: The inclination of a line towards the positive direction of x -axis.



* SPECIAL NOTE:

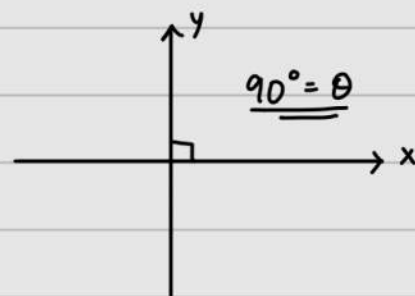
i. Slope of x -Axis $\Rightarrow 0$

ii. Slope of y -axis $\Rightarrow m = \tan 0$

$$m = \tan 90$$

$$m = \frac{1}{0}$$

$$m = \infty$$



iii. Let there be 2 lines L_1 and L_2 with slope m_1 and m_2 respectively
& if $m_1 = m_2$ then both the lines L_1 & L_2 are parallel to each other

AND

If $m_1 \cdot m_2 = -1$ then both the lines are perpendicular to each other.

* METHODS OF FINDING SLOPE

1.) When the end points of a line is given:

Let AB be a line with coordinates

A (x_1, y_1) ————— B (x_2, y_2)

Then slope of line AB = $\frac{y_2 - y_1}{x_2 - x_1}$

2.) When angle is given:

$$m = \tan \theta$$

where θ is the angle with the +ve direction of x -axis.

3.) When equation of the line is given

Let AB be a line whose equation is $Ax + By + C = 0$, then:

$$m = \frac{-A}{B}$$

Increasing
and
Decreasing

* Increasing / Decreasing function :-

It is also known as monotonic function

- a function $f(x)$ is said to be increasing at $x=a$; if $f'(x) \geq 0$
- a function $f(x)$ is said to be decreasing at $x=a$; if $f'(x) \leq 0$
- a function $f(x)$ is said to be strictly increasing at $x=a$; if $f'(x) > 0$
- a function $f(x)$ is said to be strictly decreasing at $x=a$; if $f'(x) < 0$

* Intervals :-

- Close Intervals = $[a, b]$ $\rightarrow$ all the values b/w a and b ,
including a and b
- Open intervals = (a, b) $\rightarrow$ all the values b/w a and b ,
 $]a, b[$ not including a and b .
- close open = $[a, b)$ $\rightarrow$ all the values b/w a and b ,
interval $]a, b[$ including a but not including b .
- open close = $(a, b]$ $\rightarrow$ all the values b/w a and b ,
interval $]a, b]$ including b but not including a

Maxima
minima

* Maxima and Minima:~

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$$f'(x) = 3x^2 - 12x + 9$$

equate with 0

$$3x^2 - 12x + 9 = 0$$

$$(3x-3)(x-3) = 0$$

$$x = 1 \text{ or } 3$$

$$x = 1 \longrightarrow f''(x) < 1 \quad (\text{point of maxima})$$

$$x = 3 \longrightarrow f''(x) > 1 \quad (\text{point of minima})$$

NOTE: When two variables are unknown try to incorporate trigonometric values.

(When $\sin \theta$ & $\cos \theta$ is derived put $2 \sin \theta \cos \theta = \sin 2\theta$)

Shapes	Perimeter	Curve surface Area	Total surface area	Volume
• Square	$4s$	—	s^2	s^3
• Rectangle	$2(L+B)$	—	$L \times B$	$L \times B \times H$
• Cone	—	$\pi r l$	$\pi r l + \pi r^2$	$\frac{1}{3} \pi r^2 h$
• Sphere	—	—	$4\pi r^2$	$\frac{4}{3} \pi r^3$
• Hollow cylinder	—	—	$2\pi r h$	$\pi r^2 h$
• Closed cylinder	—	—	$2\pi r h + 2\pi r^2$	$\pi r^2 h$
• Cylinder open in one side	—	—	$2\pi r h + \pi r^2$	$\pi r^2 h$

Rate
measure

* Rate measure

take the basic x and y equation



differentiate it



rate of change in $x = \frac{dx}{dt}$; in $y = \frac{dy}{dt}$



put $\frac{dx}{dt} / \frac{dy}{dt}$ in place of x and y differentiates



then solve to find

Indefinite integration

* Formulae:-

$$1. \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$2. \int \frac{dx}{x} = \log |x| + c$$

$$3. \int dx = x + c$$

$$4. \int a^x dx = \frac{a^x}{\log a} + c$$

$$5. \int e^x dx = e^x + c$$

$$6. \int e^{-x} dx = -e^{-x} + c$$

$$7. \int x dx = \frac{x^2}{2} + c$$

$$8. \int k dx = kx + c \quad \{k \text{ is constant}\}$$

$$9. \int \sin x dx = -\cos x + c$$

$$10. \int \cos x dx = \sin x + c$$

$$11. \int \tan x dx = \log |\sec x| + c$$

or

$$-\log |\cos x| + c$$

$$12. \int \cot x dx = \log |\sin x| + c$$

$$13. \int \operatorname{cosec} x \, dx = \log |\operatorname{cosec} x - \cot x| + c$$

$$14. \int \sec x \, dx = \log |\sec x + \tan x| + c$$

$$15. \int \sec^2 x \, dx = \tan x + c$$

$$16. \int \operatorname{cosec}^2 x \, dx = -\cot x + c$$

$$17. \int \sec x \tan x \, dx = \sec x + c$$

$$18. \int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$$

$$19. \int \frac{dx}{1+x^2} = \tan^{-1} x + c$$

$$20. \int \frac{-dx}{1+x^2} = \cot^{-1} x + c$$

$$21. \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$$

$$22. \int \frac{-dx}{\sqrt{1-x^2}} = \cos^{-1} x + c$$

$$23. \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$$

$$24. \int \frac{-dx}{x\sqrt{x^2-1}} = \operatorname{cosec}^{-1} x + c$$

* Types of Integration

→ Using substitution

→ Using Trigonometrical transformation

- $1 - \cos 2u = 2 \sin^2 u$
- $1 + \cos 2u = 2 \cos^2 u$
- $\sin 2u = 2 \sin u \cos u$
- $2 \cos A \cos B = \cos (A+B) + \cos (A-B)$
- $2 \sin A \sin B = \cos (A-B) - \cos (A+B)$
- $2 \sin A \cos B = \sin (A+B) + \sin (A-B)$
- $2 \cos A \sin B = \sin (A+B) - \sin (A-B)$

→ Integration by parts:—

$$\int u v \, dv$$

$$u \int v \, dv - \int \left[\frac{d}{du} u \times \int v \, dv \right] du$$

I = Inverse T.

L = log

A = Algebraic

T = Trigo

E = Exponent

→ Integration in the form

$$\int e^u \{ f(u) + f'(u) \} du$$

$$= e^u f(u) + c$$

→ Some special Integrals
(only to be put in algebraic pattern questions)

$$\bullet \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left(\frac{x-a}{x+a} \right) + c$$

$$\bullet \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left(\frac{a+x}{a-x} \right) + c$$

$$\bullet \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

→ Integration in the form $\int \frac{dx}{ax^2 + bx + c}$

$x^2 \rightarrow$ coefficient should be free if not, take common and make it free.

[make shift denominator in $x^2 + 2cx + c^2$ then turn into $(x+c)^2$]

→ Integration in the form $\int \frac{Px + Q}{ax^2 + bx + c}$

(bring the differentiation of denominator to numerator

$$\int \frac{Px + k}{ax^2 + bx + c} \quad \swarrow + \quad \int \frac{Q - k}{ax^2 + bx + c}$$

↓

$\log |ax^2 + bx + c|$

↓

solve like the previous method. + c

→ Integration in the form

$$\int \frac{dx}{a+b \sin x} \quad / \quad \int \frac{dx}{a \cos x + b} \quad / \quad \int \frac{dx}{a \sin x + b \cos x} \quad / \quad \int \frac{x^2 + a^2}{x^4 + a^4}$$

→ Integration in the form

$$\int \frac{dx}{a + b \sin^2 x} \quad / \quad \int \frac{dx}{a \cos^2 x + b} \quad / \quad \int \frac{dx}{a \sin^2 x + b \cos^2 x}$$

① divide numerator and denominator by $\cos^2 x$

② Convert the denominator in terms of $\tan$.

③ put $\tan x = t$

→ put more special Integrals

$$① \int \frac{dx}{\sqrt{x^2 + a^2}} = \log |x + \sqrt{x^2 + a^2}| + c$$

$$② \int \frac{dx}{\sqrt{x^2 - a^2}} = \log |x + \sqrt{x^2 - a^2}| + c$$

$$③ \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$$

$$④ \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + c$$

$$\textcircled{5} \quad \int \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + c$$

$$\textcircled{6} \quad \int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$$

→ Partial fraction:—

$A, B, C, D \longrightarrow \text{constant}$

If in denominator

corresponding partial fraction

- $(x-a)$ $\frac{A}{(x-a)}$
- $(x-a)(x-b)$ $\frac{A}{(x-a)} + \frac{B}{(x-b)}$
- $(x-a)(x-b)(x-c)$ $\frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$
- $(x-a)(x^2+b)$ $\frac{A}{(x-a)} + \frac{Bx+C}{(x^2+b)}$
- $(x^2+a)(x^2+b)$ $\frac{Ax+B}{(x^2+a)} + \frac{Cx+D}{(x^2+b)}$
- $(x-a)^2$ $\frac{A}{(x-a)} + \frac{B}{(x-a)^2}$
- $(x-a)^3$ $\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-a)^3}$

definite Integration

$$* \int e^u \{ f(u) + f'(u) \} + c$$

$$\Rightarrow e^u \{ f(u) \} + c$$

$$* \int \frac{du}{u^2 - a^2} = \frac{1}{2a} \log \left| \frac{u-a}{u+a} \right| + c$$

$$* \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \log \left| \frac{a+u}{a-u} \right| + c$$

$$* \int \frac{du}{u^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + c$$

* when in the form $\int \frac{du}{au^2 + bu + c}$ → Split the denominator into $(u^2 - a^2)$ or $(a^2 - u^2)$ or $(u^2 + a^2)$
 then use the above powers.

* when in the form $\frac{Pn + Q}{au^2 + bu + c}$

bring the diff of denominator on numerator.

$$* \text{ when in the form } \int \frac{u^2 + a^2}{u^4 + a^4} du$$

= divide num & denominator by u^2

$$(a-b)^2 + 2ab = a^2 + b^2$$

$$\text{OR } a^2 + b^2 = (a+b)^2 - 2ab.$$

(If the numerator has (-ve) sign then break denom into $(a+b)^2 - 2ab$ and vice versa).

* Integration in the form

$$\int \frac{du}{a+b\sin^2 u} \quad \text{or} \quad \int \frac{du}{a\cos^2 u+b} \quad \text{or} \quad \int \frac{du}{a\sin^2 u+b\cos^2 u}$$

- div num & den. by $\cos^2 u$.
- convert denominator to tan.
- put $\tan u = t$

* When in the form

$$\int \frac{du}{a+b\sin u} \quad \text{or} \quad \int \frac{du}{a\cos u+b} \quad \text{or} \quad \int \frac{du}{a\sin u+b\cos u}$$

- write $\sin u = \frac{2\tan \frac{u}{2}}{1+\tan^2 \frac{u}{2}}$ & $\cos u = \frac{1-\tan^2 \frac{u}{2}}{1+\tan^2 \frac{u}{2}}$

* Special Integrals

- $\int \frac{du}{\sqrt{u^2+a^2}} = \log |u+\sqrt{u^2+a^2}| + c$

- $\int \frac{du}{\sqrt{u^2-a^2}} = \log |u+\sqrt{u^2-a^2}| + c$

- $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1} \frac{u}{a} + c$

- $\int \sqrt{u^2+a^2} du = \frac{u}{2} \sqrt{u^2+a^2} + \frac{a^2}{2} \log |u+\sqrt{u^2+a^2}| + c$

- $\int \sqrt{u^2-a^2} du = \frac{u}{2} \sqrt{u^2-a^2} - \frac{a^2}{2} \log |u+\sqrt{u^2-a^2}| + c$

- $\int \sqrt{a^2-u^2} du = \frac{u}{2} \sqrt{a^2-u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + c$

Differential equation

* Differential Equation:-

- dependent variable and Independent variable

$$y = x^2 + 5x + 6$$

$$\text{when } x = 0, y = 6$$

here, x is independent &

$$x = 1, y = 12$$

y is dependent variable

An equation containing an independent variable, a dependent variable and derivatives of dependent variable is called a differential equation.

* Order of differential equation

The order of highest order derivative occurring in a differential equation is called order.

* Degree of differential equation

The power of the highest order derivative occurring in a differential equation after it is made free from radical and fraction.

eg. $\left(\frac{d^2y}{dx^2}\right)^3 + 4\left(\frac{dy}{dx}\right)^4 = 6y$

$$\text{Order} = 2 \quad \text{degree} = 3$$

* Types of solutions:-

- ↳ general solution
- ↳ Particular solution

$$(eg. y = A \sin x + B \cos x)$$

$$(eg. \frac{d^2y}{dx^2} + y = 0)$$

NOTE: • No. of arbitrary constant in a general solution of differential equation of order 4? $\Rightarrow \underline{4}$

• No. of arbitrary constant in a particular solution of differential equation of order 2? $\Rightarrow \underline{0}$

* Variable separable equation:-

- ↳ first identify $\frac{dy}{dx}$ then separate and solve

* linear differential equation:-

$$\frac{dy}{dx} + py = q$$

$$IF = e^{\int p dx}$$

$$y \times IF = \int (IF \times q) dx$$

$$\frac{dx}{dy} + px = q$$

$$IF = e^{\int p dy}$$

$$x \times IF = \int (IF \times q) dy$$

* homogenous differential equation

$\frac{dy}{dx} \rightarrow$ divide by the highest power of x (both numerator and denominator)

$\frac{dx}{dy} \rightarrow$ divide by the highest power of y (both num and deno)

then take x/y or y/x as v .

NOTE: • $c = -c$ (doesn't matter)

• $-\log y = \log y^{-1}$ ** very important

Probability

* Probability (chances)

- Sample Space

$n(s) \rightarrow$ All total possible outcome

- Event

$$n(E) \leq n(s)$$

$$P(E) = \frac{n(E)}{n(s)}$$

$$0 \leq P(E) \leq 1$$

- With replacement and Without Replacement

In with replacement, sample space remains same for the second round & in without replacement sample space is reduced for the second round.

- * When more than one item is drawn at once, combination is used. Use Combination for without Replacement (IF POSSIBLE)

$${}^nC_r = \frac{n!}{(n-r)! \times r!}$$

- Odds in Favour

If odds in favour of the event is $m:n$ then probability of occurrence of the event that is $P(E) = \frac{m}{m+n}$ and non-occurrence

of the event that is $P(\bar{E}) = \frac{n}{m+n}$

- Odds against the favour

If odd against the event is $p:q$ then probability of occurrence of the event that is $P(E) = \frac{q}{p+q}$ and non occurrence of the event

that is $P(\bar{E}) = \frac{p}{p+q}$

- Compound Probability

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2. $P(A - B) = P(A \cap \bar{B}) = P(A) - P(A \cap B)$

3. $P(B - A) = P(B \cap \bar{A}) = P(B) - P(A \cap B)$

4. $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$\cup$ = Union, atleast one of the event, OR, either, addition

$\cap$ = intersection, atleast, and, common, both, multiply

$A - B \Rightarrow$ only A

$B - A \Rightarrow$ only B

$\bar{A} \cap \bar{B} \Rightarrow$ neither A nor B

- Mutually Exclusive Event

If A and B are two events associated with a random experiment and if $P(A \cup B) = P(A) + P(B)$, then A & B are said to be mutually exclusive event.

Note: If $A \cap B = \emptyset$, then also A & B are exclusive event

- Mutually Exhaustive Event

If A, B, C are 3 events associated with a random experiment if $P(A) + P(B) + P(C) = 1$, then A, B, C are said to be exhaustive event.

- Independent Event

Let A & B are two events associated with random experiment & occurrence of A does not depend on occurrence of B then A & B are said to be independent events.

In terms of probability A & B are to be independent event if $P(A \cap B) = P(A) \times P(B)$

- Conditional Probability

a. $P(A/B) = \frac{P(A \cap B)}{P(B)}$

b. $P(B/A) = \frac{P(A \cap B)}{P(A)}$

c. $P(\bar{A}/B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

d. $P(A/\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{P(A) - P(A \cap B)}{1 - P(B)}$

e. $P(\bar{A}/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = \frac{1 - P(A \cup B)}{1 - P(B)}$

$A/B \rightarrow$ Occurrence of event A given that B has already occurred.

* BAYE'S THEOREM

It is also called inverse probability

FORMAT: let E_1 be the event of producing from M_1
let E_2 " " M_2
let E_3 " " M_3

let H denote defective

$$\begin{array}{lll} P(E_1) = \underline{\hspace{2cm}} & P(E_2) = \underline{\hspace{2cm}} & P(E_3) = \underline{\hspace{2cm}} \\ P(H/E_1) = \underline{\hspace{2cm}} & P(H/E_2) = \underline{\hspace{2cm}} & P(H/E_3) = \underline{\hspace{2cm}} \end{array}$$

$$P(E_2/H) = \frac{P(E_2) \times P(H/E_2)}{P(E_1) \times P(H/E_1) + P(E_2) \times P(H/E_2) + P(E_3) \times P(H/E_3)}$$

* Probability distribution

The collection of probability of a particular event is called probability distribution.

NOTE: The sum of the probability of the probability distribution is always equal to one.

* Mean and Variance

Mean is also called average/expected value written as $E(x)$.

NOTE: Mean > Variance (always in random variable)

$$\text{Mean} = \sum np$$

$$\text{Variance} = \sum n^2 p - (\sum np)^2$$

Application of differentiation in commerce and economics

* formulae:-

- $Tc = \text{Total Cost} = C(x) = TFC + TVC$

$$\begin{aligned} C(x) &= \frac{3x^2 + 5x + 7}{x} \\ x=1 &= \frac{8}{1} + 7 \\ x=2 &= \frac{22}{2} + 7 \\ &\quad \downarrow \quad \downarrow \\ &\quad TVC \quad TFC \end{aligned}$$

- $AC = \text{Average cost} = \frac{Tc}{x} \frac{(\text{Total cost})}{(\text{output})}$

slope means differentiation

- $MC = \text{Marginal Cost} = \frac{d}{dx} Tc$

- $TR = \text{Total Revenue} = p(x) \times x$ [demand function $\times$ quantity]

- $AR = \frac{TR}{x}$

- $MR = \frac{d}{dx} TR$

- $\text{Profit} = TR - TC$

- $\text{loss} = TC - TR$

- $\text{No profit, no loss} = TC = TR$

* Names:-

- $p(x) = \text{demand function}$

- $C(x) = \text{cost function}$

- $R(x) = \text{revenue function}$

- $P(x) = \text{profit function}$

* Properties:-

$$i. \int_a^b f(x) dx = \int_a^b f(t) dt$$

$$ii. \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$* iii. \int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$$

[use this property blindly for modulus functions].

for eg: [in case of modulus sums]

$$\int_1^3 f(x) dx, \text{ where } f(x) = |x-2|$$

$$x-2=0$$

$$x=2$$

$$\left[-(x-2) \right]_1^2 \quad \left[(x-2) \right]_2^3$$

$$*** iv. \int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

$$*** v. \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$* vi. \int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx; \text{ even} \\ 0; \text{ odd} \end{cases}$$

NOTE: It is even when $f(-x) = f(x)$
and odd when $f(-x) = -f(x)$

[whenever there is log take it as I and solve using the v property mainly]

$$vii. \int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx; & f(2a-x) = f(x) \\ 0, & f(2a-x) = -f(x) \end{cases}$$

Linear regression

* Regression:-

Independent Variable

Dependent Variable

$$\underset{\substack{\downarrow \\ \text{dependent (y)}}}{y} = \frac{x^2 + 2x + 5}{\substack{\downarrow \\ \text{independent (x)}}}$$

There are various methods of solving regression eqⁿ:

- ① Method of least square / linear trend method / straight line method.

There are 2 regression eqⁿ:

- ① y is dependent on x (y on x)
- ② x is dependent on y (x on y)

y on x

regression eqⁿ

y on x

$$\boxed{y = a + bx}$$

a and b are constant
which can be found using
2 normal eqⁿ.

↓

$$\begin{aligned}\sum y &= na + b\sum x \\ \sum xy &= a\sum x + b\sum x^2\end{aligned}$$

x on y

regression eqⁿ

x on y

$$\boxed{x = a + by}$$

a and b are constant
which can be found
using 2 normal eqⁿ.

↓

$$\begin{aligned}\sum x &= na + b\sum y \\ \sum xy &= a\sum y + b\sum y^2\end{aligned}$$

- Regression Coefficient :-

Let $y = a + bx$ is the regression eqⁿ of y on x . Then the regression coeff of y on x denoted as $b_{yx} = b$. It is also called slope of y on x

Let $x = a + by$ is the regression eqⁿ of x on y . Then the regression coeff of x on y denoted as $b_{xy} = b$. It is also called slope of x on y

Methods of finding Regression Coefficient :-

(i). When x and y is given :-

$$y \text{ on } x : b_{yx} = \frac{\text{cov}(x, y)}{\text{var. } x}$$

cov = co-variance

var = Variance

$$x \text{ on } y : b_{xy} = \frac{\text{cov}(x, y)}{\text{var. } y}$$

$$\text{cov}(x, y) = \frac{\sum xy}{n} - \frac{\sum x \cdot \sum y}{n^2}$$

$$\text{Variance } x = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n} \right)^2$$

$$\text{Variance } y = \frac{\sum y^2}{n} - \left(\frac{\sum y}{n} \right)^2$$

$$\text{variance} = (\text{standard deviation})^2$$

Special note: $r = \sqrt{b_{xy} \times b_{yx}}$, where r is correlation coefficient and its value lies b/w $-1 \leq r \leq 1$ always.

if $r = 0.8$, b_{ny} , b_{yn} is -ve, then units are as -0.8.

- $\frac{\text{Mean of } x}{\bar{x}}$ and $\frac{\text{Mean of } y}{\bar{y}}$

If 2 regression eqⁿ are given and we are asked to find mean of x and mean of y , solve both the regression eqⁿ simultaneously and the value of x and y so obtained is mean of x and mean of y .

NOTE: if $r = \pm 1$, then, the 2 lines of regression coincide with one another.

if $r = 0$, then, both the line of regression are ⊥ to each other or // to co-ordinate axis.

Is b/w 2 lines:

$$\tan \theta = \left| \frac{1 - r^2}{b_{yn} + b_{ny}} \right|$$

Linear programming

* Linear Programming

NOTE: To shade the region we put $x=0$ and $y=0$ (always) and if condition is satisfied shade towards the origin and if condition is not satisfied shade against the origin.

[at least means greater than equal to
at most means less than equal to]